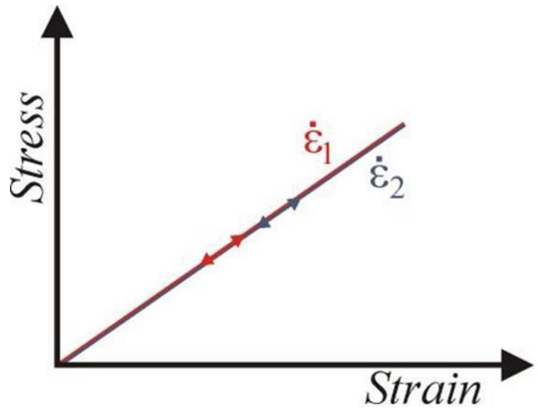
# BE172 Week 6: Viscoelasticity

Lab goals: experimentally measure viscoelastic properties of tissue, and estimate material properties with a simple mathematical model

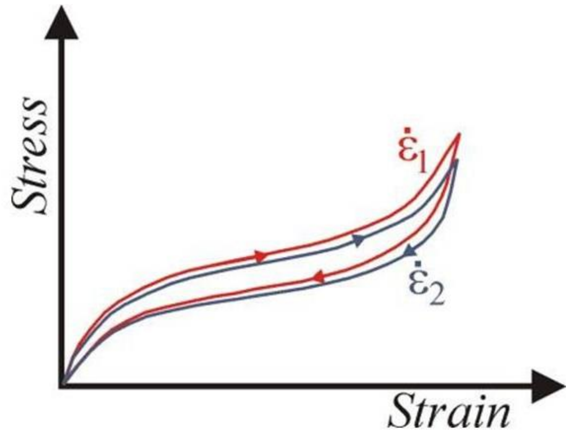
Viscoelastic material: has properties of both a solid and fluid

Experimentally seen as hysteresis, creep and stress relaxation

A purely elastic material would not have viscoelastic properties




elastic

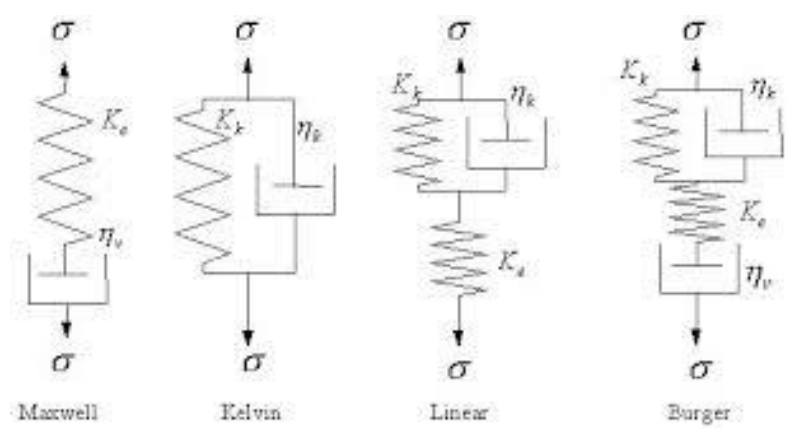



viscoelastic

Overall goals:

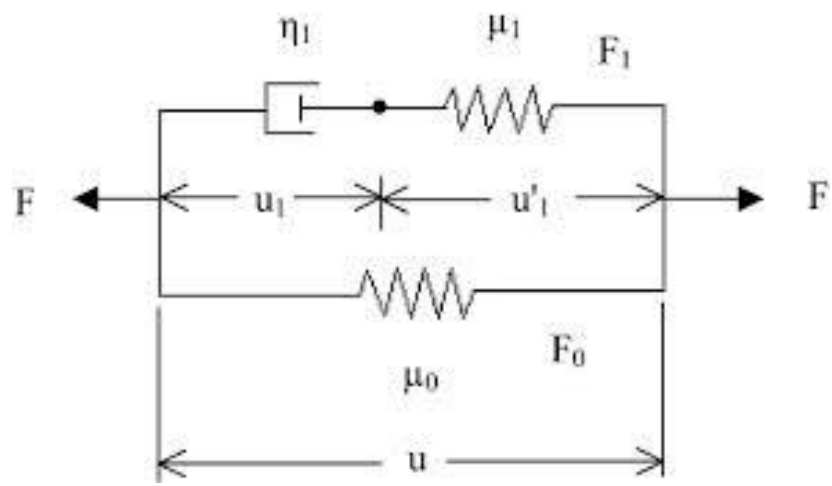
- Stress relaxation and creep tests on fresh (frozen) rat tissue
  record force and length on computers vs. time
- Determine exponential constants of response
- Use 3-element model to determine viscoelastic parameters of tissues.

## Mathematical models of viscoelastic materials:




Models with elastic (spring) and viscous (dashpot) elements

# Kelvin 3-element model (Standard linear solid)




- Spring: F = kx or F = $\mu u$
- Dashpot (viscous element): F = ηv or F = $\eta\dot{u}$

$u$ = displacement, μ = spring constant, $\dot{u}$ = velocity, η = viscosity coefficient (Fung notation)

# By combining components, can derive the governing equation for a Kelvin model

Force-displacement relationship:

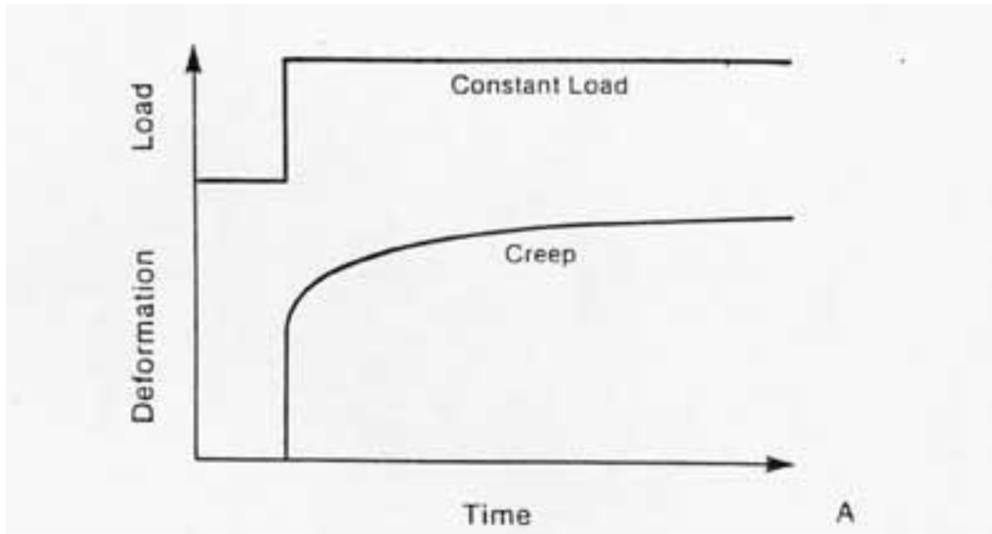
$$F + \frac{\eta}{\mu_1}\dot{F} = \mu_o u + \eta\left(1 + \frac{\mu_o}{\mu_1}\right)\dot{u}$$

In terms of time constants:

$$F + \tau_\varepsilon \dot{F} = E_R(u + \tau_\sigma \dot{u})$$

$$\tau_\varepsilon = time\ constant\ for\ strain\ (\varepsilon) = constant$$
$$\tau_\sigma = time\ constant\ for\ stress\ (\sigma) = constant$$
$$E_R = relaxed\ elastic\ modulus$$

# Creep test: apply a constant step force (stress)

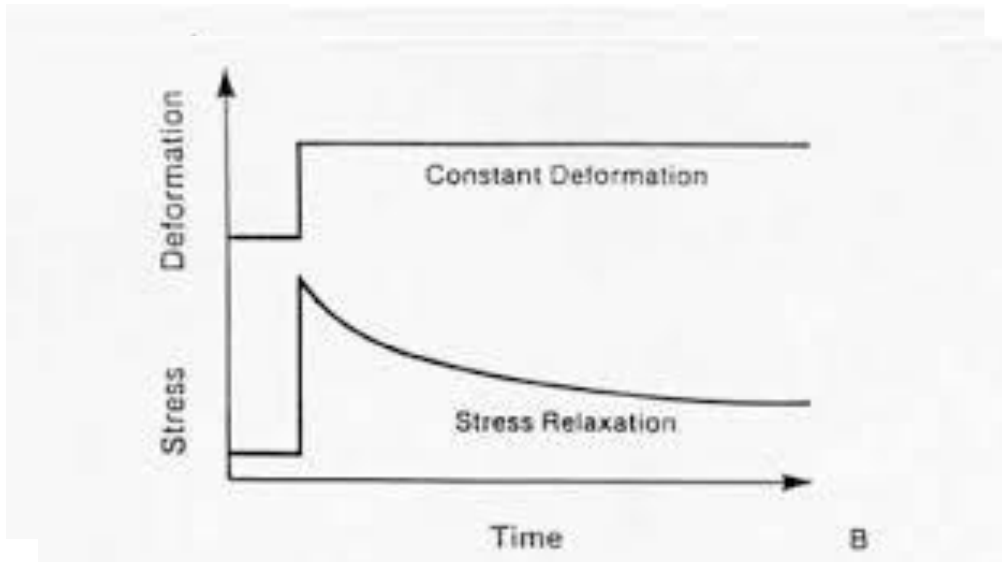



Mathematical solution is a “creep function”:

$$C(t) = \frac{1}{E_R}\left[1 - \left(1 - \frac{\tau_\epsilon}{\tau_\sigma}\right) e^{-t/\tau_\sigma}\right] \mathrm{F(0)}$$

- Initial deformation (displacement) depends on amount of load [F(0)]
- Tissue will continue to displace (creep) reaching and asymptotic level
- Can measure time constant of this creep ($\tau_\sigma$)

# Stress relaxation test: apply a constant step displacement




In this case we have a decaying exponential "relaxation function":

$$K(t) = E_R \left[1 - \left(1 - \frac{\tau_\sigma}{\tau_\epsilon}\right) e^{-t/\tau_\varepsilon}\right] u(0)$$

- Similar experimental set up, but control length, measure change in force
- For either exponential response, can fit with proper function to exponential time constant
- Measure ($\tau_\varepsilon$) from this experiment

# Constants from measured curves

**Time constants (τ):**

Curve fit to measured decaying or rising exponential. Can use Sigmaplot
For curve fitting, or other computer software.

Make sure to pick the proper function form, and use appropriate data:

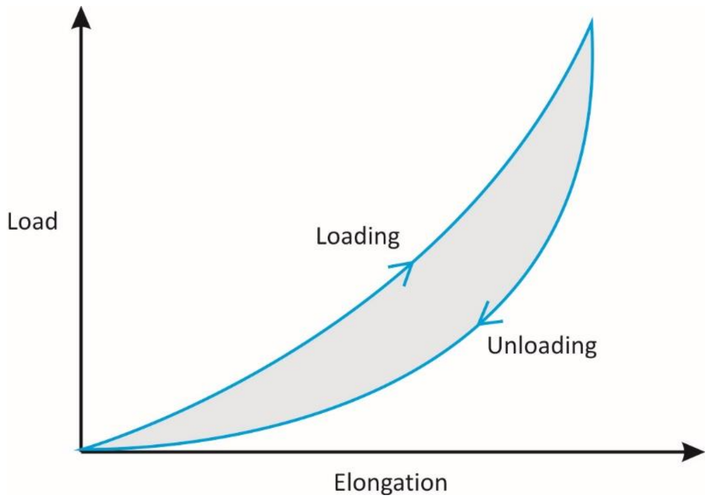
Can't fit data such as:


With only a decaying exponential (Blue line)

**Reduced relaxation modulus (ER):**

Assume t=∞, can find $E_R$ from both tests

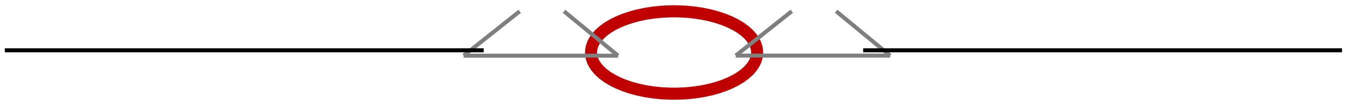
# Hysteresis: measure area between loading and unloading force-displacement curves




- Normalize hysteresis values between different specimens

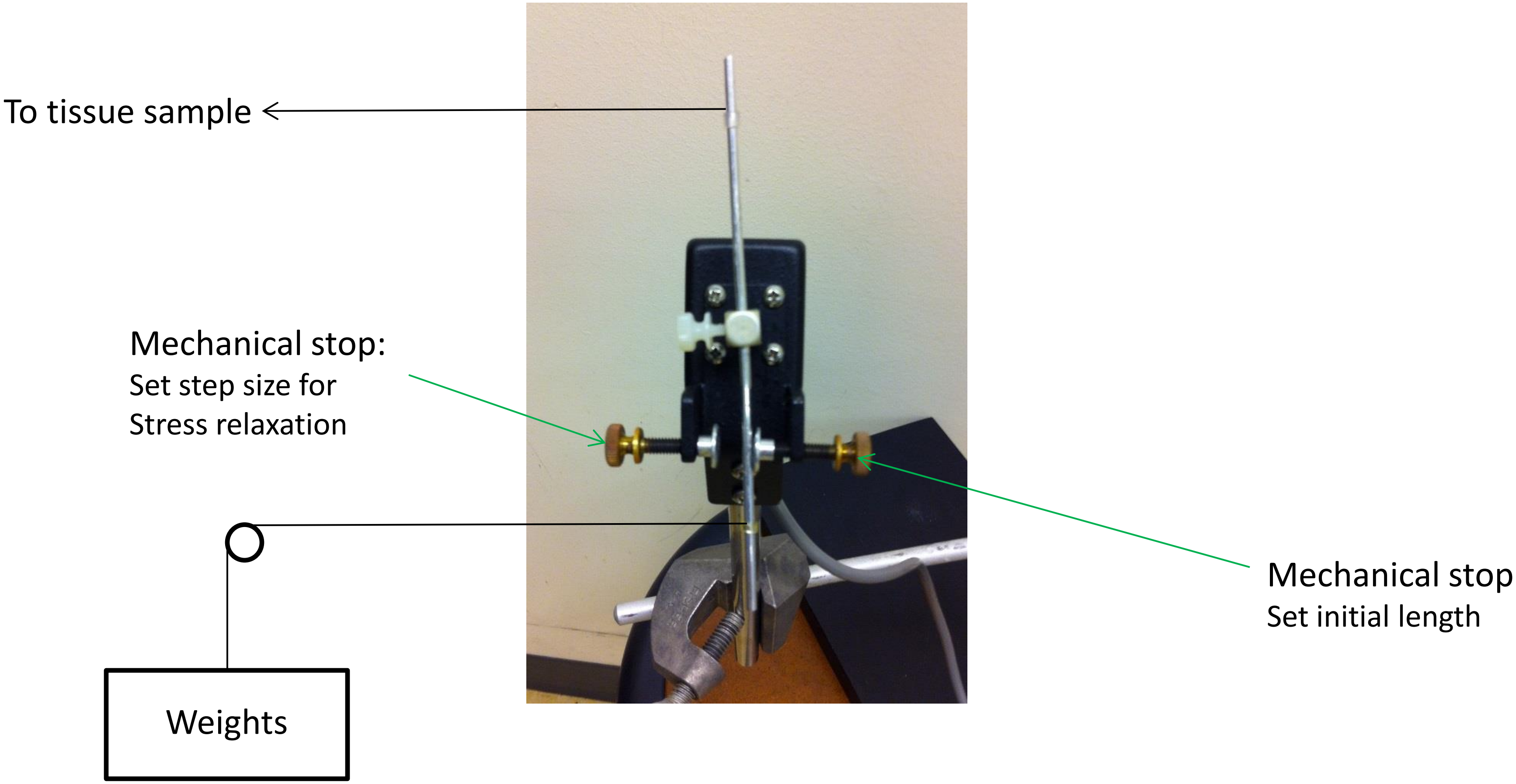
# Experiment highlights

- Small specimens (2-3 mm diameter) from rats
- Connect specimens to suture with staples:



- Set up force-displacement acquisition similar to force velocity setup
- Use computer for all data acquisition (calibrate with o-scope)
- Test system with rubber band sample, practice all 3 types of tests

# Same isotonic “displacement” gauge




- Move left mechanical stop for creep test: allow step change in force via weights

# Lab report uploading of electronic copy:

- As part of ongoing BE ABET accreditation, we are requiring electronic uploading of Labs 6 and 10 this quarter.
- See link on TritonEd for uploading each of the 2 reports
- Please upload a PDF copy of your lab, in addition to turning in the hard copy
- You will only receive a score on the lab only if you upload the PDF!